Semiclassical corrections to the large-N limit of Dicke's model

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We use a semiclassical expansion as an alternative derivation of the well-known, rigorous result obtained by Hepp and Lieb for the classical limit of the spin-boson model. We also explicitly derive correction terms to the classical limit previously obtained in the context of Heisenberg equations of motion. We analyze the size and shape of the N (number of atoms) vs t (time) domain whithin which the corrections so obtained are useful.

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I. INTRODUCTION

The spin-boson model [1] is a paradigm in several areas of physics such as statistical mechanics [2–4], condensed matter physics [5,6], quantum optics [7–11], and more recently the theory of decoherence [12–14] and quantum chaos [15–17]. Also, on a more formal side, the model belongs to a class of models called mean field models for which the classical limit is mathematically well defined. The first proof of its classical limit, given by Hepp and Lieb [18,19] and later complemented and clarified in [20], was given in the framework of Heisenberg equations of motion.

The purpose of the present contribution is to derive that result in Schrödinger's picture by means of a semiclassical expansion of the time-dependent equation. Although not as rigorous, the semiclassical expansion allows us to calculate quantum corrections to the classical limit for the expection values of operators. It has been shown to be convergent in a specific model in [21].

The idea behind the semiclassical expansion we use is the following. The essentially quantum character of the Heisenberg equations of motion for operators becomes apparent in the correlation functions. On the other hand, quantum effects become apparent in a two-degree-of-freedom system, e.g., in their entanglement, a property of the *state* of the system. In order to perturbatively access the entanglement between the spin and boson generated by their interaction, we set up the semiclassical approximation such that its zeroth-order contribution is a product state and contains all ingredients of the classical dynamics. The corrections carry quantum effects, in particular, entanglement. We use this expansion to calculate the same expectation values considered in [18,20] and obtain quantum corrections to them, shown to be of order N^{-1} as expected, N being the number of atoms, a characteristic size of the spin-boson system.

In Sec. II we review the main results of the derivation in [18,20]. In Sec. III we derive the semiclassical approximation, which is a generalization of [21], now for two degrees of freedom. Moreover, the expectation values of operators are obtained to second order in the semiclassical expansion. Conclusions are given in Sec. IV.

II. THE SPIN-BOSON MODEL AND ITS CLASSICAL LIMIT

The Hamiltonian of the spin-boson model describing N two-level atoms subject to an electromagnetic field of frequency ω is given (in units of $\hbar\omega$) by

$$H_N = \epsilon J_z + a^\dagger a + \frac{G}{\sqrt{N}}(J_+ + J_-)(a + a^\dagger), \tag{1}$$

where the operators a and $a^\dagger$ are the standard bosonic creation and destruction operators and the real constants ϵ and G are the ratio between the level transition frequency and the frequency of the field and the coupling constant, respectively. The operators $J_\pm$ and J_z are collective mode spin operators [1] obtained as follows:

$$J_{x,y,z} = \sum_{j=1}^{N} J^j_{x,y,z},$$

$$J^j_{+,-,z} = 1 \otimes 1 \otimes \cdots \otimes 1 \otimes \sigma^j_{+,-,z} \otimes 1 \otimes \cdots \otimes 1, \tag{2}$$

where $\sigma_{+,-,z}$ are Pauli matrices.

The classical limit of the model is obtained with the following "intensive" operators:

$$\tilde{J}_{+,-,z} = \frac{J_{+,-,z}}{N}, \quad \tilde{a} = \frac{a}{\sqrt{N}}, \quad \tilde{a}^\dagger = \frac{a^\dagger}{\sqrt{N}}. \tag{3}$$

By means of standard commutation relations with the Hamiltonian in Eq. (1), it is easy to get equations of motion for these operators:

$$\dot{\tilde{J}}_+ = i[\epsilon\tilde{J}_+ - 2G\tilde{J}_z(\tilde{a} + \tilde{a}^\dagger)], \tag{4}$$

$$\dot{\tilde{J}}_- = -i[\epsilon\tilde{J}_- - 2G\tilde{J}_z(\tilde{a} + \tilde{a}^\dagger)], \tag{5}$$

$$\dot{\tilde{J}}_z = -iG(\tilde{J}_+ - \tilde{J}_-)(\tilde{a} + \tilde{a}^\dagger), \tag{6}$$

$$\dot{\tilde{a}} = -i[\tilde{a} + G(\tilde{J}_+ + \tilde{J}_-)], \tag{7}$$

$$\dot{\tilde{a}}^\dagger = i[\tilde{a}^\dagger + G(\tilde{J}_+ + \tilde{J}_-)]. \quad (8)$$

One of these operators, describing the density of photons $\tilde{a}^\dagger\tilde{a}$, was used by Wang and Hioe [2] to describe the phase transition of the model in the thermodynamic limit.

Another important concept in the description of the classical limit of Hepp and Lieb is that of a "classical state." Let $\{A_N^i\}_{i=1}^5$ represent the intensive operators defined above. For a given density operator χ^N acting on the Hilbert space of H_N, suppose we can take the limit

$$\lim_{N\to\infty} \mathrm{Tr}\{\chi^N A_N^i\} = \alpha^i \quad (9)$$

for a complex number α^i. This density operator χ^N represents a classical state with respect to the intensive operators $\{A_N^i\}$ with value α_i if, in addition to the existence of the above limit, we also have

$$\lim_{N\to\infty} \mathrm{Tr}\{\chi^N (A_N^i - \alpha^i)^\dagger (A_N^i - \alpha^i)\} = 0. \quad (10)$$

Equation (10) says that the variance of an intensive operator should vanish in the limit $N\to\infty$. Generalized coherent states [22] are examples of realizable classical states and they will be in fact the main subject of this paper. Coherent states of the harmonic oscillator were used by Glauber in the description of the radiation field [23]. For this case—the harmonic oscillator or h_4 algebra—the coherent states are eigenvectors of the bosonic destruction operator a and they are labeled by their eigenvalue, that is, for a coherent state $|x\rangle$ we have

$$a|x\rangle = x|x\rangle, \quad (11)$$

where x is a complex number. Field coherent states are obtainable through the action of the displacement operator

$$D(x) \equiv \exp(xa^\dagger - x^* a) \quad (12)$$

on the vacuum Fock state $|0\rangle$,

$$|x\rangle = D(x)|0\rangle. \quad (13)$$

Atomic coherent states were introduced by Agarwal [24]. They are obtainable with the action of an "atomic" displacement operator

$$D(y) \equiv \exp\left(\frac{\arctan|y|}{y}(yJ_+ - y^* J_-)\right) \quad (14)$$

on an extremal state of the J_z basis,

$$|y\rangle = D(y)|j, \pm j\rangle. \quad (15)$$

The $|j,-j\rangle$ state is the usual choice and will also be ours. As in the harmonic oscillator case, the label y is a complex number. Here, however, the label is not an eigenvalue. To see how coherent states can represent a member of the class of classical states, consider the state given by a product of coherent states,

$$|\alpha,y\rangle \equiv D(\sqrt{N}x) \otimes D(y)|0\rangle|j,-j\rangle. \quad (16)$$

By the action of the displacement operators on the operators of their respective algebra and using $N=2j$, it is easy to obtain

$$\langle\alpha,y|\tilde{a}|\alpha,y\rangle = x, \quad (17)$$

$$\langle\alpha,y|\tilde{a}^\dagger|\alpha,y\rangle = x^*, \quad (18)$$

$$\langle\alpha,y|\tilde{J}_+|\alpha,y\rangle = \frac{y^*}{1+|y|^2} \equiv \langle\tilde{J}_+\rangle, \quad (19)$$

$$\langle\alpha,y|\tilde{J}_-|\alpha,y\rangle = \frac{y}{1+|y|^2} \equiv \langle\tilde{J}_-\rangle, \quad (20)$$

$$\langle\alpha,y|\tilde{J}_z|\alpha,y\rangle = -\frac{1}{2}\left(\frac{1-|y|^2}{1+|y|^2}\right) \equiv \langle\tilde{J}_z\rangle. \quad (21)$$

Equations (17)–(21) just show that the limit on (9) is well defined for the state $\chi^N = |\sqrt{N}x, y\rangle\langle\sqrt{N}x, y|$. We can also check that this state satisfies condition (10). For the field degree of freedom we have

$$D^\dagger(\sqrt{N}x)\tilde{a}D(\sqrt{N}x) = \tilde{a} + x, \quad (22)$$

so the variance of this operator goes to zero trivially. For the atomic degree of freedom we may write after some algebra

$$\langle y|(\tilde{J}_z - \langle\tilde{J}_z\rangle)^\dagger(\tilde{J}_z - \langle\tilde{J}_z\rangle)|y\rangle = \frac{1}{N}\frac{|y|^2}{1+|y|^2}, \quad (23)$$

$$\langle y|(\tilde{J}_+ - \langle\tilde{J}_+\rangle)^\dagger(\tilde{J}_+ - \langle\tilde{J}_+\rangle)|y\rangle = \frac{1}{N}\frac{|y|^4}{(1+|y|^2)^2}, \quad (24)$$

$$\langle y|(\tilde{J}_- - \langle\tilde{J}_-\rangle)^\dagger(\tilde{J}_- - \langle\tilde{J}_-\rangle)|y\rangle = \frac{1}{N}\frac{|y|^4}{(1+|y|^2)^2}. \quad (25)$$

That is, the variance goes to zero when $N\to\infty$.

Up to this point we have just stated important definitions and showed a concrete example of a classical state. We may now state the theorem by Hepp and Lieb [18].

If χ^N is classical with respect to $\{\tilde{a},\tilde{a}^\dagger,\tilde{J}_z,\tilde{J}_+,\tilde{J}_-\}$ at the point $\vec{r}=\{\langle\tilde{a}\rangle,\langle\tilde{a}^\dagger\rangle,\langle\tilde{J}_z\rangle,\langle\tilde{J}_+\rangle,\langle\tilde{J}_-\rangle\}$, the state evolved according to the Hamiltonian (1), $\chi^N(t)=e^{-iHt}\chi^N(0)e^{iHt}$, will also be classical at the point $\vec{r}(t) =\{\langle\tilde{a}\rangle(t),\langle\tilde{a}^\dagger\rangle(t),\langle\tilde{J}_z\rangle(t),\langle\tilde{J}_+\rangle(t),\langle\tilde{J}_-\rangle(t)\}$, and the evolution of $\vec{r}(t)$ is given by the equations

$$\langle\dot{\tilde{J}}\rangle_+ = i[\epsilon\langle\tilde{J}_+\rangle - 2G\langle\tilde{J}_z\rangle(\langle\tilde{a}\rangle + \langle\tilde{a}^\dagger\rangle)], \quad (26)$$

$$\langle\dot{\tilde{J}}_-\rangle = -i[\epsilon\langle\tilde{J}_-\rangle - 2G\langle\tilde{J}_z\rangle(\langle\tilde{a}\rangle + \langle\tilde{a}^\dagger\rangle)], \quad (27)$$

$$\langle\dot{\tilde{J}}_z\rangle = -iG(\langle\tilde{J}_+\rangle - \langle\tilde{J}_-\rangle)(\langle\tilde{a}\rangle + \langle\tilde{a}^\dagger\rangle), \quad (28)$$

$$\langle\dot{\tilde{a}}\rangle = -i[\langle\tilde{a}\rangle + G(\langle\tilde{J}_+\rangle + \langle\tilde{J}_-\rangle)], \quad (29)$$

$$\langle\dot{\tilde{a}}^\dagger\rangle = i[\langle\tilde{a}^\dagger\rangle + G(\langle\tilde{J}_+\rangle + \langle\tilde{J}_-\rangle)]. \quad (30)$$

III. A SEMICLASSICAL APPROXIMATION IN THE SPIN-BOSON MODEL

The classical counterpart $\mathcal{H}$ of the quantum Hamiltonian is defined by a projection of H in coherent states, that is, for this two-degree-of-freedom model,

$$\mathcal{H}_N \equiv \langle x,y|H_N|x,y\rangle = -N\frac{\epsilon}{2}\left(\frac{1-|y|^2}{1+|y|^2}\right) + |x|^2 + \frac{NG(y+y^*)(x+x^*)}{\sqrt{N}(1+|y|^2)}, \tag{31}$$

where $|x,y\rangle \equiv |x\rangle \otimes |y\rangle$ is a direct product of coherent states and x and y represent the two labels of the first and second degrees of freedom. $\mathcal{H}(x,y)$ generates a classical dynamics for the two labels x and y. This classical dynamics will have the following equations of motion for the labels:

$$\frac{dx}{dt} = -i\frac{\partial \mathcal{H}_N}{\partial x^*} = -i\left(x + \frac{NG(y+y^*)}{\sqrt{N}(1+|y|^2)}\right), \tag{32}$$

$$\frac{dy}{dt} = -i\frac{(1+|y|^2)^2}{N}\frac{\partial \mathcal{H}_N}{\partial y^*} = -i\left(\epsilon y + \frac{G(1-y^2)(x+x^*)}{\sqrt{N}}\right), \tag{33}$$

$$\frac{dx^*}{dt} = i\frac{\partial \mathcal{H}_N}{\partial x} = i\left(x^* + \frac{NG(y+y^*)}{\sqrt{N}(1+|y|^2)}\right), \tag{34}$$

$$\frac{dy^*}{dt} = i\frac{(1+|y|^2)^2}{N}\frac{\partial \mathcal{H}_N}{\partial y} = i\left(\epsilon y^* + \frac{G(1-y^{*2})(x+x^*)}{\sqrt{N}}\right). \tag{35}$$

The dynamics corresponding to the Hamiltonian $\mathcal{H}_N(\sqrt{N}x,y)$ is equivalent to that of $\mathcal{H}_1(x,y)$

To calculate the corrections in the classical limit of the spin-boson model, we will set up a semiclassical approximation and try to build quantum dynamics through classical ingredients. The semiclassical approximation is constructed as follows. We choose $H_N^{\rm sc}$ such that

$$\mathcal{H}_N = \langle x,y|H_N^{\rm sc}|x,y\rangle \tag{36}$$

and the time evolution under $\mathcal{H}_N^{\rm sc}$ does not change the character of a state which is initially a product of coherent states. Also, the evolution of the state, apart from a global phase, is given by the evolution of the labels as in Eqs. (32) and (35). The semiclassical expansion is now defined in terms of these ingredients. Consider $H_N = H_N^{\rm sc} + \Delta$, where $\Delta = H_N - H_N^{\rm sc}$ can be treated as a perturbation. Use the interaction picture to obtain

$$|\Psi^I(t)\rangle = \left(1 - i\int_0^t dt_1\Delta(t_1) - \int_0^t\int_0^{t_1} dt_1 dt_2 \Delta(t_1)\Delta(t_2) + \cdots\right) \times |x_0,y_0\rangle, \tag{37}$$

where $\Delta(t) = U_N^{\rm sc\dagger}(t)\Delta U_N^{\rm sc}(t)$, $|x_0,y_0\rangle$ is a product of coherent states with labels x_0 and y_0, and $U_N^{\rm sc}(t)$ satisfies

$$\dot{U}_N^{\rm sc}(t) = -\frac{i}{\hbar}H_N^{\rm sc}U_N^{\rm sc}(t). \tag{38}$$

Notice that the zeroth order of the expansion contains all elements of the classical dynamics and is therefore "as classical as possible" in the sense that entanglement between the field and atom degrees of freedom is contained in the correction terms.

For this particular model, the semiclassical Hamiltonian can be defined as

$$H_N^{\rm sc} = a^\dagger a + \epsilon J_z + \frac{G}{\sqrt{N}}[(a^\dagger - x^*)\langle J_-\rangle + (a-x)\langle J_+\rangle + x^* J_- + xJ_+] + \frac{G}{\sqrt{N}}[(a^\dagger - x^*)\langle J_+\rangle + (a-x)\langle J_-\rangle + x^*J_+ + xJ_-], \tag{39}$$

where $\langle J_\pm\rangle = \langle y|J_\pm|y\rangle$, the mean value of $J_\pm$ in the atomic coherent state with label y, and $x = \langle x|a|x\rangle$ is the mean value of the operator a in the electromagnetic field coherent state with label x. $U_N^{\rm sc}$ is easily seen to be given by

$$U_N^{\rm sc}(t) = D_1(x)\exp[i\omega(t)a^\dagger a + \phi(t)]D_1^{-1}(x_0) \otimes D_2(y)\exp[i\nu(t)J_z]D_2^{-1}(y_0), \tag{40}$$

where x_0 and y_0 are the labels of the initial coherent states, x and y are given by the classical trajectories through $\mathcal{H}(x,y)$, and we can find the temporal dependence of the remaining parameters ω, ν, ϕ by substituting (40) in (38). We get

$$\omega(t) = -t, \tag{41}$$

$$\dot{\nu} = -\epsilon + \frac{G}{\sqrt{2j}}(xy^* + x^*y) + \frac{G}{\sqrt{2j}}(x^*y^* + xy), \tag{42}$$

$$\dot{\phi} = \frac{j[G(xy^* + x^*y) + G(xy + x^*y^*)]}{\sqrt{2j}(1+|y|^2)}. \tag{43}$$

To first order in Δ we get for Dicke's model

$$|\Psi^I(t)\rangle = \frac{|x_0,y_0\rangle + c_{11}(t)|x_1,y_1\rangle}{[1+|c_{11}(t)|^2]^{1/2}}, \tag{44}$$

where $|x_1,y_1\rangle$ are generalized coherent states obtained by the action of the displacement operator on the corresponding first excited states: $|x_1,y_1\rangle = D_1(x_0)\otimes D_2(y_0)|1;j,-j+1\rangle$. The coefficient c_{11} is independent of N and it is obtained from the equation

$$\dot{c}_{11}(t) = -i\frac{G[1-y^2(t)]}{\sqrt{2j}[1+|y(t)|^2]}\exp\{i[t-\nu(t)]\}. \tag{45}$$

Note that the quantum correction is given in terms of classical trajectories and a phase containing the classical action. Now, at second order, we can use

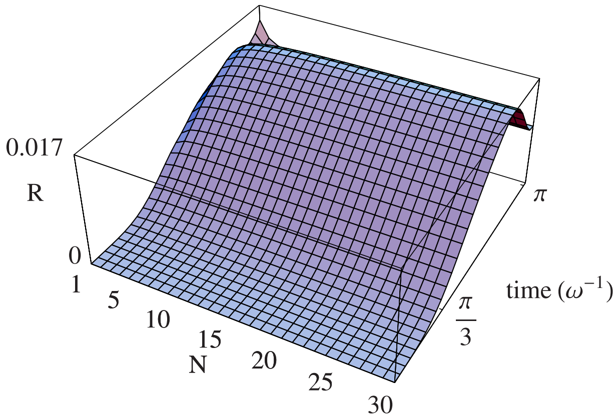

FIG. 1. (Color online) Ratio $R(t)$ defined in Eq. (61) for several values of particle number N. $\epsilon=1$, $G=0.2$, $x_0=0.404\,748$, and $y_0=-0.070\,888\,1$. Time is in units of ω^{-1}; all other constants are dimensionless.

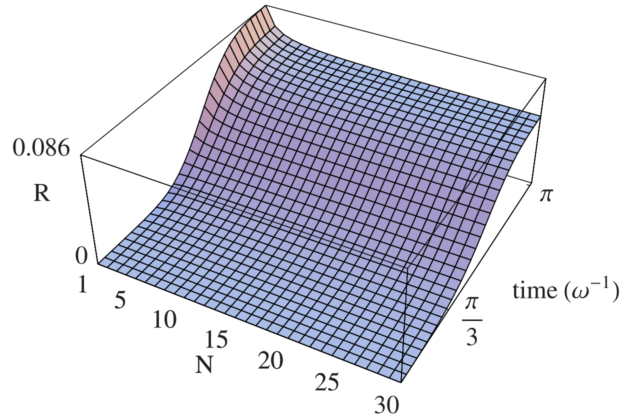

FIG. 2. (Color online) Ratio $R(t)$ defined in Eq. (61) for several values of particle number N. $\epsilon=1$, $G=0.2$, $x_0=0.070\,710\,7(1+i)$, and $y_0=0.071\,066\,9(1+i)$. Time is in units of ω^{-1}; all other constants are dimensionless.

$$|\Psi^I(t)\rangle^{(n)} = \int_0^t d\tau\,\Delta(\tau)|\Psi^I(\tau)\rangle^{(n-1)}, \tag{46}$$

where $|\Psi^I(t)\rangle^{(n)}$ stands for the nth-order correction. $|\Psi^I(t)\rangle$ is always of the form

$$|\Psi^I(t)\rangle = A^{-1}\left((1+c_{00})|x_0,y_0\rangle + \sum_{(k+l)>1} c_{kl}(t)|x_k,y_l\rangle\right), \tag{47}$$

where we have $|x_k,y_l\rangle = D(x)\otimes D(y)|k,l\rangle$. Up to second order we have

$$\dot{c}_{00}(t) = \frac{-ic_{11}(t)G[1-y(t)^{*2}]e^{-it+i\nu(t)}}{[1+|y(t)|^2]}, \tag{48}$$

$$\dot{c}_{01}(t) = \frac{ic_{11}(t)2G[y(t)^*+y(t)]e^{-it}}{(\sqrt{N}[1+|y(t)|^2])}, \tag{49}$$

$$\dot{c}_{02}(t) = \frac{-ic_{11}(t)\sqrt{2(N-1)}G[1-y(t)^2]e^{-it-i\nu(t)}}{\sqrt{N}[1+|y(t)|^2]}, \tag{50}$$

$$\dot{c}_{20}(t) = \frac{-ic_{11}(t)\sqrt{2}G[1-y(t)^{*2}]e^{it+i\nu(t)}}{[1+|y(t)|^2]}, \tag{51}$$

$$\dot{c}_{21}(t) = \frac{ic_{11}(t)2\sqrt{2}G[y(t)+y(t)^*]e^{it}}{\sqrt{N}[1+|y(t)|^2]}, \tag{52}$$

$$\dot{c}_{22}(t) = \frac{-ic_{11}(t)2\sqrt{N-1}G[1-y(t)^2]e^{it-i\nu(t)}}{\sqrt{N}[1+|y(t)|^2]}. \tag{53}$$

The coefficient A is just a normalization. We are now in a position to show that the present perturbative series reproduces the result obtained in [18] and corrections to that result. For this purpose we consider the time evolution of expectation values of the form

$$\langle\hat{O}\rangle = \langle\alpha(t),y(t)|\hat{O}|\alpha(t),y(t)\rangle, \tag{54}$$

where we have a classical state given by the coherent state

$$|\alpha(t),y(t)\rangle \equiv D[\sqrt{N}x(t)]\otimes D[y(t)]|0\rangle\otimes|j,-j\rangle. \tag{55}$$

Using first- and second-order semiclassical approximations, it is just a matter of straightforward calculation to determine the mean of these intensive operators. We get, for $\tilde{a}$ up to second order,

$$\langle\tilde{\alpha}\rangle(t) = x(t) + \frac{1}{A\sqrt{N}}(c_{11}c_{01}^* + c_{21}c_{11}^*); \tag{56}$$

for $\tilde{J}_z$ up to second order,

$$\langle\tilde{J}_z\rangle(t) = \frac{1}{2}\left(\frac{1-|y(t)|^2}{1+|y(t)|^2}\right) + \frac{y(t)[(1+c_{00})c_{01}^* + c_{20}c_{21}^*] + \sqrt{2}(c_{01}c_{02}^* + c_{21}c_{22}^*)}{A\sqrt{N}[1+|y(t)|^2]} + \text{c.c.}; \tag{57}$$

for $\tilde{a}^2$,

$$\langle\tilde{\alpha}^2\rangle(t) = x(t)^2 + \frac{1}{AN}\{2(c_{11}c_{01}^* + c_{21}c_{11}^*) + \sqrt{2}[c_{22}c_{02}^* + c_{21}c_{01}^* + c_{20}(1+c_{00})^*]\}; \tag{58}$$

for $\tilde{J}_z^{\,2}$ the corrections already appear at first order,

$$\langle\tilde{J}_z^{\,2}\rangle(t) = \frac{1}{4}\left(\frac{1-|y(t)|^2}{1+|y(t)|^2}\right)^2 + \left(\frac{[1-|y(t)|^2]^2|c_{11}|^2 + |y(t)|^2(1+3|c_{11}|^2)}{N(1+|c_{11}|^2)[1+|y(t)|^2]^2}\right); \tag{59}$$

and the interaction terms in the Hamiltonian of the model are corrected as follows:

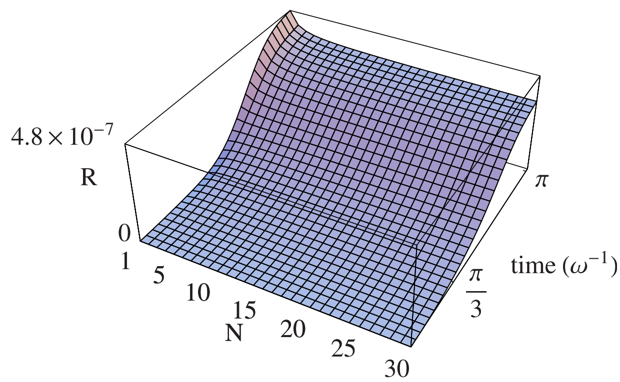

FIG. 3. (Color online) Ratio $R(t)$ defined in Eq. (61) for several values of particle number N. $\epsilon=1$, $G=0.01$, $x_0=0.070\,710\,7(1+i)$ and $y_0=0.071\,066\,9(1+i)$. Time is in units of ω^{-1}; all other constants are dimensionless.

$$\langle\tilde{a}\tilde{J}_+\rangle = \frac{x(t)y^*(t)}{2[1+|y(t)|^2]} + \frac{x(t)}{A\sqrt{N}}\{(1+c_{00})c_{01}^* + c_{20}c_{21}^* + \sqrt{2}(c_{01}c_{02}^* + c_{21}c_{22}^*) - y^*(t)[c_{01}(1+c_{00})^* + c_{21}c_{20}^*] - \sqrt{2}y^*(t)(c_{02}c_{01}^* + c_{22}c_{21}^*)\}, \tag{60}$$

with similar expressions for the other terms.

IV. RESULTS AND CONCLUSION

In the above equations, the coefficients c_{ij} have their temporal dependence given by (48)–(53) and the variables $x(t)$ and $y(t)$ obey the classical equations given by the Hamiltonian (36). We recognize $x(t)$ and $[1-|y(t)|^2]/\{2[1+|y(t)|^2]\}$ as the means of the operators $\tilde{a}$ and $\tilde{J}_z$, respectively. Moreover, the first term on the right-hand side of Eqs. (56)–(60) agree with the result obtained by Hepp and Lieb [Eq. (10) here]. These correction terms are the main result of this contribution, and we readily see that they are of order $1/\sqrt{N}$ or $1/N$.

Note that the coefficients $c_{ij}(t)$ depend on both N and t, although this is not explicit. One may now ask about the adequacy of these correction terms, i.e., is there an N-t domain within which the corrections terms behave as such? A way to answer the proposed question is to look at the size and shape of the N-t domain where the correction terms are small enough. We should keep in mind that this domain is dependent on the initial coherent state. Since all coefficients, except $1+c_{00}$, are related to semiclassical corrections, a suggestive way to look for a classical regime would be through the ratio

$$R(t) \equiv \frac{\Sigma|c_{kl}(t)|}{|1+c_{00}(t)|}. \tag{61}$$

Perhaps the most interesting result is that for small values of N, the quantum corrections show a strong dependence on N (all other variables fixed). However, as N grows, $R(t)$ tends to a "universal" curve which is independent of N, although it still depends on the initial state. Also, for times such that $t<\omega^{-1}$, the corrections do not depend on N.

Our findings are illustrated as follows. Figures 1 and 2 show the dependence of $R(t)$ on the initial condition and also the large-N behavior. Figures 3 and 4 exhibit the dependence of $R(t)$ on the coupling constant. Note that, since $G=0.5$ corresponds to the well-known phase transition coupling, the validity of the expansion is limited to very small values of t.

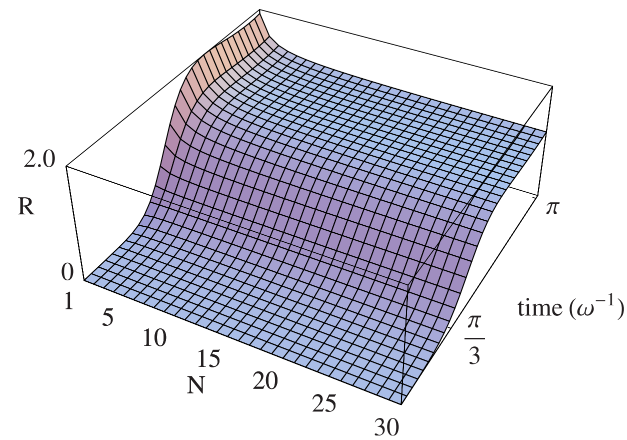

FIG. 4. (Color online) Ratio $R(t)$ defined in Eq. (61) for several values of particle number N. $\epsilon=1$, $G=0.5$, $x_0=0.070\,710\,7(1+i)$, and $y_0=0.071\,066\,9(1+i)$. Time is in units of ω^{-1}; all other constants are dimensionless.

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